You Think. . .You Got Problems

Effective problem solving skills are needed to successfully navigate the collegiate and workplace environments. Cultivating good problem solving skills requires preparation, practice and persistence. Polya's four step problem solving process first published in *How to Solve It* is a powerful and adaptable strategy and will serve as a starting point in developing problem solving skills.

Polya's Four Step Process

1. Understand the Problem
2. Devise a Plan
3. Carry out the Plan
4. Check results

These simple but powerful steps that can be applied to almost any problem, real life, arithmetic, algebra, word problems or virtually any other situation to build a solid problem solving repertoire. Many people disregard the first and last steps, not gaining a thorough understanding of the problem, jumping right in to find an answer and never looking back and are frequently stumped by similar problems later. Serious reflection on the problems and their solutions with a focus on the problem solving process is precisely what is needed to truly develop into a good problem solver.

1. Understanding the Problem/Preparation

It is important to fully understand the problem to be solved and *keep in mind exactly what is to be found or done*. Problems can be presented in a variety of ways; orally or in various written formats such as words, charts, tables or diagrams. You must understand the vocabulary used in the context of the problem at hand and should become familiar with typical problems and formats specific to your area of study. You will need to develop reading and listening strategies for extracting information, determining what is known and what is to be found, as well as what information is extraneous or unnecessary.

Ask yourself:

- Can you restate the problem in your own words?
- What do the key words *really* mean?
- What *exactly* is the problem asking you to do?
- Do you have the necessary information?
- Do you need all the given information?
- Would a diagram or picture help?

- How will you know when you are finished?
- What will the answer look like?

Things to try/do:

- ✓ Put the problem in context (restate and relate to the real world)
- ✓ State specifically the goal of the problem
- ✓ Make a list or table of given information, including units
- ✓ Disregard extraneous information
- ✓ Draw a picture or diagram and represent the unknown
- ✓ Make a model (mental, written or physical)

2. <u>Devise a plan/Incubation</u>

Think about the problem as you understand it. In planning a solution, the key is to relate the given information to the task at hand. Consider modeling a solution after a similar or related problem, there is an advantage to having a large "bag of tricks". Perhaps the problem can be solved as a series of sub-problems or by breaking it down into smaller parts. The focus of this stage is to *connect what you know to what must be done*. Making this connection may involve mathematical operations, formulas or equations, and may require additional research. For more involved problems a complete method of solution not usually obvious. Try to think "outside the box", take a chance on a fresh approach and explore the possibilities.

Ask yourself:

- Do you know a related or similar problem?
- Would a formula be useful in solving the problem?
- Do I have all the necessary information, if not can I find it?
- Can the problem be broken into smaller parts?
- How does what I know help me toward my goal?

Things to try/do:

- ✓ Gather additional information using recall, estimation or research
- ✓ Think of the problem as partially solved
- ✓ Solve a simpler related problem
- ✓ Eliminate extraneous information
- ✓ Work backwards
- ✓ Guess check and revise

3. Carry out the Plan/Illumination – In some respects the easiest of the four steps and the most revealing. Carefully carry out the plan, checking the work and logic at every step. Working through a problem can give insight into how things work and add to your "bag of tricks". Should carrying out the original plan prove difficult, go back and devise another plan of attack using what has been learned from the first attempt. Persistence is the name of the game here.

Ask yourself:

- Is this right?
- Can it be proved to be correct?

Things to do:

- Keep an organized record of your work
- Double check each step or sub-problem
- Reevaluate your plan as you work (possibly repeat step 2)

4. Check Results/Verification – Look back on the solution and the problem solving process. The solution should be reasonable, actually answer the question posed and have the correct units. The process used to arrive at the solution could lead to solving additional problems in the future, consider what other problems could be solved in a similar way. Reflect back on the problem solving *process* for effectiveness and possibly generalize the results.

Ask yourself:

- Do the results make sense?
- Does the result have the expected units?
- Do the results compare favorably with your estimate?
- Can you solve it differently and get the same result?
- Can other problems be solved in this same way?
- Is the solution surprising?

Things to do
:

- Verify the solution meets the conditions of the problem
- Retrace your steps and reexamine what you have done
- Represent your solution clearly and concisely
- Consider any uncertainties or assumptions made
- Consider the implications of your results

Activities and Problems

Students ideally will practice solving problems in the context of their vocational education program. The applications should be directly relevant to their course of study. This will minimize the challenge of transferring applied math skills to their coursework and the workplace.

Vocabulary activity (List course specific vocabulary)
Make a list of words that are used in a special way related to your discipline or course of study.
Example: gross and net, wages

Journal Create a library of specific math problems (and solutions) encountered in your discipline

1. Compute the gross weekly earnings of an employee that makes $10.25 per hour and works a 40 hour week. (Level 3)

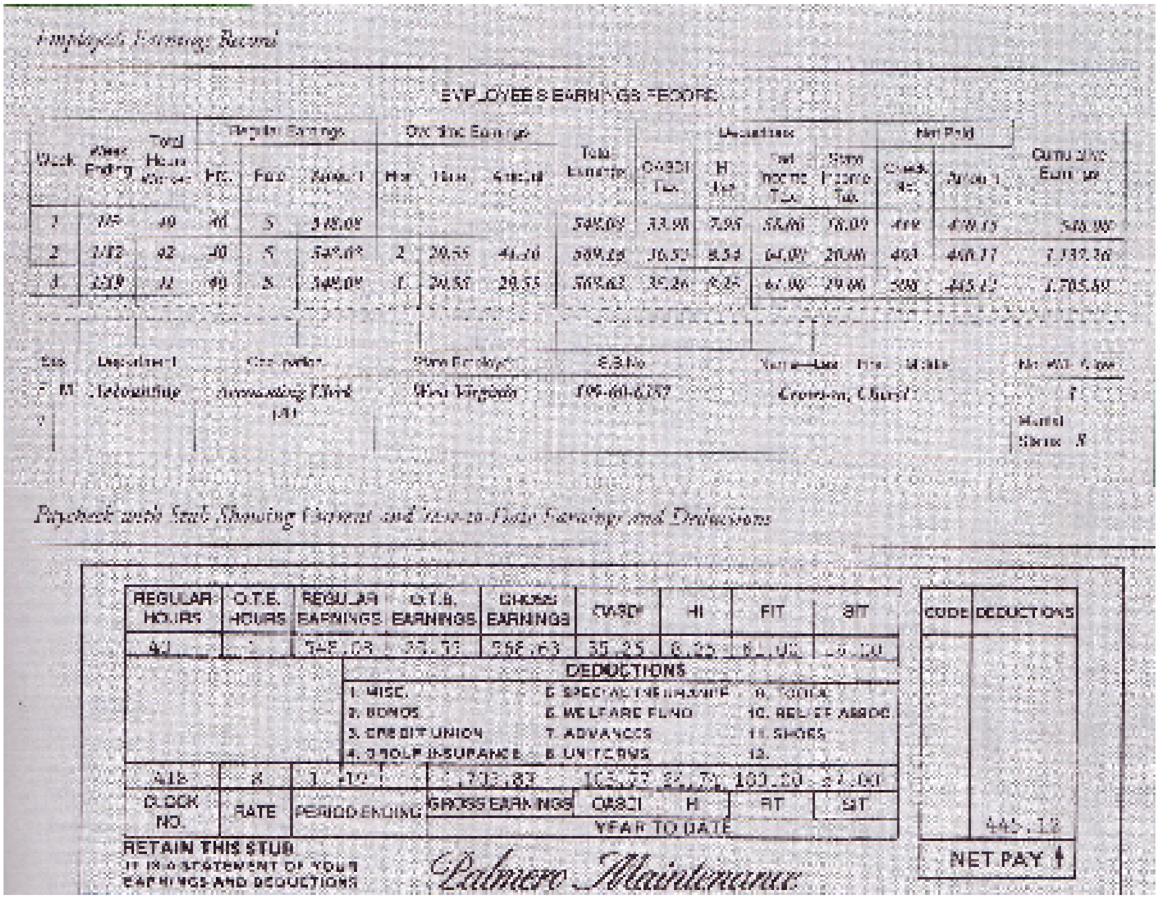
2. Compute the total gross weekly earnings of an employee that works a 40 hour week at regular time and 7 hours of overtime. The employee makes $ 9.50 per hour regularly, and time and a half for overtime. (Level 4)

3. Refer to the given time card:

Compute Ralph Pope's total gross weekly earnings if his regular hourly wage is $11.00 per hour. (Level 4)

Employee Weekly Hours **Hours worked over 8.0 in a day paid at 1.5 of regular hourly rate.**							
Employee	**M**	**TU**	**W**	**TH**	**F**	**S**	**Total Hours**
Ralph Pope	8.0	6.0	9.0	7.0	10.0		
Lynn Priest	7.0	3.0	7.0	8.0	8.0	4.5	

The following diagrams are to be used with example questions 4 through 8:



Employee's Earnings Record

EMPLOYEE'S EARNINGS RECORD

Week	Week Ending	Total Hours Worked	Regular Earnings: Hrs.	Rate	Amount	Overtime Earnings: Hrs.	Rate	Amount	Total Earnings	OASDI Tax	HI Tax	Fed. Income Tax	State Income Tax	Net Paid: Check No.	Amount	Cumulative Earnings
1	1/5	40	40	5	548.08				548.08	33.98	7.95	58.00	18.09	418	430.15	548.08
2	1/12	42	40	5	548.08	2	20.55	41.10	589.18	36.53	8.54	64.00	20.00	463	460.11	1,137.26
3	1/19	41	40	5	548.08	1	20.55	20.55	568.63	35.26	8.25	61.00	29.00	508	445.12	1,705.89

Sex	Department	Occupation	State Employed	S.S. No.	Name—Last, First, Middle	No. W/H Allow.
F	Accounting	Accounting Clerk	West Virginia	199-60-6357	Crowson, Cheryl	1

Marital Status S

Paycheck with Stub Showing Current and Year-to-Date Earnings and Deductions

REGULAR HOURS	O.T.E. HOURS	REGULAR EARNINGS	O.T.E. EARNINGS	GROSS EARNINGS	OASDI	HI	FIT	SIT
40	1	548.08	20.55	568.63	35.25	8.25	61.00	19.00

DEDUCTIONS

1. MISC. 5. SPECIAL INSURANCE 9. TOOLS
2. BONDS 6. WELFARE FUND 10. RELIEF ASSOC.
3. CREDIT UNION 7. ADVANCES 11. SHOES
4. GROUP INSURANCE 8. UNIFORMS 12.

CLOCK NO.	RATE	PERIOD ENDING	GROSS EARNINGS	OASDI	HI	FIT	SIT
418	8	1-19-[illegible]	1,705.83	105.77	24.74	183.00	67.00

YEAR TO DATE

CODE	DEDUCTIONS
	445.12

NET PAY

RETAIN THIS STUB
IT IS A STATEMENT OF YOUR EARNINGS AND DEDUCTIONS

Palmero Maintenance

4. What percentage of Cheryl's gross earnings was withheld from her check in week three? (Level 5)

5. Compare the withholdings in weeks one and two by amounts and as a percentage of gross earnings. (Level 5)

6. Is Cheryl's overtime rate correct, based on the federal requirement of one and a half times the hourly rate for hours worked over forty in a workweek? (Level 6)

7. Cheryl's time card for week ending 1/19/07 is shown below. Cheryl feels that she has been under paid, explain. (Level 6)

Employee Time Record
Employee Name Cheryl Crowson
Pay Type Salary
Pay period for the weekending January 19, 2007

Date	Time In	Time Out	Total Hours
1/15	8:00 am	3:30 pm	7.5 hrs
1/16	8:15 am	4:30 pm	8 hrs. 45 min.
1/17	8:15 am	4:30 pm	8 ¾ hours
1/18	8:00 am	4:15 pm	8hrs. 15 min.
1/9	8:00 am	4:45 pm	8.75 hrs

8. Create a plausible time card for Cheryl for the payroll week ending January 19, 2007. With the following constraints; Cheryl can not arrive before 9:00 am, she must take a 30 minute unpaid lunch break, and must leave when the office closes at 6:00 pm.(Level 7)

Level 3: All needed information is presented in a logical order and translates easily from a word problem to a math equation. There is no extra information given and only straight- forward, single math operations are used on whole numbers. Simple conversions (time and money) and changing numbers from one form to another may be performed.

Level 4: The information may be presented out of order and extra, unnecessary data may be included. It might be necessary to read and/or interpret a graph or diagram to glean out important, needed information. Problems may include more than one math operation, as well as, ordering the given information before performing calculations such as averages, ratios, proportions and rates using whole numbers and decimals .

Level 5: Problems require several steps of logic and calculations. Decisions must be made as to what information, calculations, or conversions are needed to solve the problem. Formulas may be needed to make conversions between systems of measurement. Comparisons may be used to determine the best choice in a given situation. Calculations of perimeters and areas of simple shapes may be required, as well as, percent discounts or markups

Level 6: Problems may require considerable translation from verbal to mathematical expression and will generally require considerable setup and involve multiple-step calculations. The use of two formulas may be needed to change from one unit in one system of measurement to a unit in another system. Formulas may need to be rearranged before they are used. Finding errors and troubleshooting mistakes in problems that belong at levels 3, 4 or 5 may be required. Finding the volume of rectangular solids and calculating multiple rates may be expected.

Level 7: The content or format of the given information may be unusual and may even be incomplete. Solutions may involve 4 or more steps of logical reasoning. Problems may have more than one unknown and could include nonlinear relationships. Conversions between systems of measurement that involve fractions, decimals, mixed numbers and percentages may be required. Calculations of multiple areas and volumes of spheres, cylinders, or cones may be necessary. Finding the best deal when several choices are available, troubleshooting errors in level 6 questions, setting up and manipulating complex ratios and proportions, applying basic statistical concepts, calculating multiple areas and volumes of spheres, cylinders, or cones may be necessary.

*Derived from ACT WorkKeys Applied Mathematics Levels

Characteristics/Skills

There are five levels of difficulty. Level 3 is the least complex and Level 7 is the most complex. The levels build on each other, each incorporating the skills assessed at the previous levels. For example, at Level 5, individuals need the skills from Levels 3, 4, and 5.

LEVEL	CHARACTERISTICS	SKILLS
3	Translate easily from a word problem to a math equation All needed information is presented in logical order No extra information	Solve problems that require a single type of mathematics operation (addition, subtraction, multiplication, and division) using whole numbers Add or subtract negative numbers Change numbers from one form to another using whole numbers, fractions, decimals, or percentages Convert simple money and time units (e.g., hours to minutes)
4	Information may be presented out of order May include extra, unnecessary information May include a simple chart, diagram, or graph	Solve problems that require one or two operations Multiply negative numbers Calculate averages, simple ratios, simple proportions, or rates using whole numbers and decimals Add commonly known fractions, decimals, or percentages (e.g., 1/2, .75, 25%) Add up to three fractions that share a common denominator Multiply a mixed number by a whole number or decimal Put the information in the right order before performing calculations
5	Problems require several steps of logic and calculation (e.g., problem may involve completing an order form by totaling the order and then computing tax)	Decide what information, calculations, or unit conversions to use to solve the problem Look up a formula and perform single-step conversions within or between systems of measurement Calculate using mixed units (e.g., 3.5 hours and 4 hours 30 minutes) Divide negative numbers Find the best deal using one- and two-

		step calculations and then comparing results Calculate perimeters and areas of basic shapes (rectangles and circles) Calculate percent discounts or markups
6	May require considerable translation from verbal form to mathematical expression Generally require considerable setup and involve multiple-step calculations	Use fractions, negative numbers, ratios, percentages, or mixed numbers Rearrange a formula before solving a problem Use two formulas to change from one unit to another within the same system of measurement Use two formulas to change from one unit in one system of measurement to a unit in another system of measurement Find mistakes in questions that belong at Levels 3, 4, and 5 Find the best deal and use the result for another calculation Find areas of basic shapes when it may be necessary to rearrange the formula, convert units of measurement in the calculations, or use the result in further calculations Find the volume of rectangular solids Calculate multiple rates
7	Content or format may be unusual Information may be incomplete or implicit Problems often involve multiple steps of logic and calculation	Solve problems that include nonlinear functions and/or that involve more than one unknown Find mistakes in Level 6 questions Convert between systems of measurement that involve fractions, mixed numbers, decimals, and/or percentages Calculate multiple areas and volumes of spheres, cylinders, or cones Set up and manipulate complex ratios or proportions Find the best deal when there are several choices Apply basic statistical concepts

Distance
1 foot = 12 inches
1 yard = 3 feet
1 mile = 5,280 feet
1 mile ≈ 1.61 kilometers
1 inch = 2.54 centimeters
1 foot = 0.3048 meters
1 meter = 1,000 millimeters
1 meter = 100 centimeters
1 kilometer = 1,000 meters
1 kilometer ≈ 0.62 miles

Area
1 square foot = 144 square inches
1 square yard = 9 square feet
1 acre = 43,560 square feet

Volume
1 cup = 8 fluid ounces
1 quart = 4 cups
1 gallon = 4 quarts
1 gallon = 231 cubic inches
1 liter ≈ 0.264 gallons
1 cubic foot = 1,728 cubic inches
1 cubic yard = 27 cubic feet
1 board foot = 1 inch by 12 inches by 12 inches

Weight
1 ounce ≈ 28.350 grams
1 pound = 16 ounces
1 pound ≈ 453.592 grams
1 milligram = 0.001 grams
1 kilogram = 1,000 grams
1 kilogram ≈ 2.2 pounds
1 ton = 2,000 pounds

Rectangle
perimeter = 2(*length* + *width*)
area = *length* x *width*

Rectangular Solid (Box)
volume = *length* x *width* x *height*

Cube
volume = (*length of side*)^3

Triangle
sum of angles = 180°
area = $\frac{1}{2}$(*base* x *height*)

Circle
number of degrees in a circle = 360°

circumference ≈ 3.14 x *diameter*
area ≈ 3.14 x (*radius*)^2

Cylinder

volume ≈ 3.14 x (*radius*)^2 x *height*

Cone

volume ≈ $\frac{3.14 \times (radius)^2 \times height}{3}$

Sphere (Ball)

volume ≈ $\frac{4}{3}$ x 3.14 x (*radius*)^3

Electricity

1 kilowatt-hour = 1,000 watt-hours
amps = watts ÷ volts

Temperature

°C = 0.56(°F – 32) or $\frac{5}{9}$(°F – 32)

°F = 1.8(°C) + 32 or ($\frac{9}{5}$ x °C) + 32